A New Argentinean Gravimetric Geoid Model: GEOIDEAR

D.A. Piñón, K. Zhang, S. Wu, and S.R. Cimbaro

Abstract

A new Argentinean gravimetric geoid model named GEOIDEAR was developed using the remove-compute-restore technique and incorporating the GOCO05S satellite-only global geopotential model (GGM) together with 560,656 land and marine gravity measurements. Terrain corrections were calculated for all gravity observations using a combination of the SRTM_v4.1 and SRTM30_Plus_v10 digital elevation models. For the regions that lacked of gravity observations, the DTU13 gravity model was utilised. The residual gravity anomalies were gridded using the tensioned spline algorithm. The resultant gravity anomaly grid was applied in the Stokes' integral using the spherical multi-band FFT approach and the deterministic kernel modification proposed by Wong and Gore. The accuracy of GEOIDEAR was assessed by comparing it with GPS-levelling derived geoid undulations at 1904 locations and the EGM2008 GGM. Results show that the new Argentinean geoid model has an accuracy of less than 10 cm.

Keywords

Argentina • Geoid • GPS-levelling • Gravity

1 Introduction

Two precise Argentinean geoid models have been published so far: GAR (Corchete and Pacino 2007) and ARG05 (Tocho et al. 2007). The first one was determined using the removecompute-restore (RCR) technique (Schwarz et al. 1990) and was based on the global geopotential model (GGM) EIGEN-GL04C (Förste et al. 2008). The gravity anomalies used in

S.R. Cimbaro Instituto Geográfico Nacional, Buenos Aires, Argentina GAR's computation were reduced to the geoid by means of the Helmert's second method of condensation (Heiskanen and Moritz 1967) and using the digital elevation models (DEMs) SRTM (Slater et al. 2006), over the land, and ETOPO2 (U.S. Department of Commerce 2006), over the oceans. The Stokes' integral in convolution form was solved applying the 1D fast Fourier transformation (FFT) technique (Haagmans et al. 1993). Finally, GAR was evaluated using 393 co-located GPS and levelling benchmarks distributed in 8 of the 23 Argentinean provinces. According to the authors, the standard deviation of the differences between the geoid heights derived from GAR and those derived from the GPSlevelling benchmarks was 21 cm.

The ARG05 geoid model was also determined using the RCR technique on the Stokes-Helmet approach. However, it was based on the EGM96 (Lemoine et al. 1998) GGM. The downward continuation was achieved using the GTOPO30 (U.S. Geological Survey 1996) DEM. ARG05 was evaluated using 539 GPS-levelling benchmarks distributed in 9 of the 23 Argentinean provinces and, according to the authors,

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the standard deviation of the differences between the geoid heights derived from ARG05 and those derived from the GPS-levelling benchmarks was 32 cm.

Since the development of GAR and ARG05, new and improved datasets became available. Regarding the GGMs, in 2009 the GOCE gravity space mission (Drinkwater et al. 2003) was launched, and therefore, a new generation of GGMs, which precisely describe the long- to mediumwavelength part of the Earth's gravity field, began to be freely distributed through the International Centre for Global Earth Models (ICGEM) website. With respect to the GPSlevelling observations, two significant improvements to the vertical component of the Argentinean GPS and spiritlevelling measurements were recently achieved: first, the geodetic reference frame POSGAR 2007 (Cimbaro et al. 2009), based on the international terrestrial reference frame ITRF 2005 (Altamimi et al. 2007) at 2006.6 epoch, was determined by the Instituto Geográfico Nacional (IGN) in 2009. Secondly, a least-squares adjustment of the spiritlevelling network in terms of geopotential numbers was performed by the IGN in 2014, and afterwards, Mader (1954) orthometric heights were determined, with a mean standard deviation of 0.058 m, for all the levelling benchmarks. Finally, in relation to the gravity observations, a new gravimetric absolute network called RAGA was established in 2014 and several gravity campaigns performed by the IGN before RAGA were reprocessed using the new absolute gravity values. Moreover, new land and marine gravity measurements collected during the last decade by many public and private agencies and universities became available.

After considering the accuracies of the aforementioned two Argentinean geoid models (i.e. GAR and ARG05) and the new available datasets (i.e. GNSS-levelling and gravity measurements, MDTs and GGMs), the IGN, together with the SPACE Research Centre of RMIT University, determined that a new precise geoid model fitted to the official vertical datum and geocentric reference frame was urgently required. This paper describes the methodology, techniques and datasets utilized for determining the new Argentinean gravimetric geoid model GEOIDEAR.

2 Computation Procedure

2.1 Helmert's Second Condensation Method

In order to obtain a geoid model using Stokes' formula there must be no masses above the surface of the geoid. Therefore, observed gravity anomalies should be given at the boundary surface (i.e. the geoid), and the topographic masses need to be removed and condensed in a thin mass layer onto the geoid (Heiskanen and Moritz 1967). In this research, the first requirement was met by applying the first-order freeair reduction (Δg_{FA}), together with the atmospheric correction, given by Hinze et al. (2005), to the observed gravity points. The land and salt water densities were considered 2.67 g cm⁻¹ and 1.03 g cm⁻¹ respectively. Finally, all the observed gravity and normal gravity values were referred to the IGSN71 gravity system (Morelli et al. 1972) and GRS80 reference ellipsoid (Moritz 1980, 2000) respectively.

The second requirement was satisfied by conducting the Helmert's second condensation method. The direct topographical effect of this method (i.e. the difference between the gravitational attraction of the actual topography and the one that has been condensed onto the geoid) was determined by applying the planar approach of the well-known Bouguer plate reduction (Heiskanen and Moritz 1967) and the linear approximation of the classical terrain correction (Moritz 1968) to the gravity anomalies. Then, the resultant gravity anomalies, usually called refined Bouguer anomalies, are given by

$$\Delta g_{RB} = \Delta g_{FA} - 2\pi G \rho H + C_T \tag{1}$$

where G is the Newtonian gravitational constant, ρ is the density of the topographic masses, and C_T is the terrain correction determined up to a distance of 166.7 km from each gravity location using the 3" resolution SRTM_v4.1 (Jarvis et al. 2008) and the 30" resolution SRTM30_Plus v10 (Becker et al. 2009) DEMs in the TC software (Forsberg 1984), which applies the rectangular prism integration method (Nagy 1966).

2.2 RCR Technique

The RCR technique is a well-known method for gravimetric geoid determination used in the Stokes-Helmert approach. It implies a spectral decomposition of the Earth's gravity field into three parts: the long-wavelength contribution from the GGM, the medium-wavelength signal from regional gravity observations and the short-wavelength part of the gravity spectrum from the topography (Zhang 1997). Therefore, geoid heights can be expressed as (Forsberg 1993)

$$N = N_{RES} + N_{IND} + N_{GGM} \tag{2}$$

where N_{RES} is the contribution from the observed gravity anomalies reduced using a GGM, N_{IND} is the indirect effect caused by a variation in the Earth's gravity potential from condensing the topographic masses onto the geoid, and N_{GGM} is the geoidal undulation derived from the spherical harmonic expansion of the GGM and it is given by (Heiskanen and Moritz 1967).

$$N_{GGM} = \frac{GM}{\gamma a} \sum_{n=2}^{\infty} \left(\frac{a}{n}\right)^n \sum_{m=0}^n \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda\right) P_{nm} \left(\cos \varphi\right)$$
(3)

where *GM* is the geocentric gravitational constant, *a* is the Earth radius, *n* and *m* are the degree and order of the spherical harmonic expansion respectively, C_{nm} and S_{nm} are the fully normalised spherical harmonics coefficients, and $P_{nm}(\cos\varphi)$ is the fully normalized associated Legendre polynomials.

2.2.1 Removing Step and Gridding of the Residual Gravity Anomalies

The first step of the RCR technique is the removal of the low-frequency part of the Earth's gravity field from Δg_{RB} . The residual gravity anomalies can be obtained from

$$\Delta g_{RES} = \Delta g_{RB} - \Delta g_{GGM} \tag{4}$$

where Δg_{GGM} is the gravity anomaly derived from the GGM and described by Heiskanen and Moritz (1967).

In this study, the GOCO05S satellite-only GGM (Mayer-Guerr 2015) was applied complete to degree and order 280 using the GEOCOL17 software (Tscherning 1985) for determining Δg_{GGM} at every gravity observation location.

Moreover, since the FFT technique was used to calculate the Stokes' integral, a regular grid generated from the discrete Δg_{RES} values was required. Due to the fact that there were many areas that lacked of land and marine gravity measurements, the world gravimetric model DTU13 (Andersen et al. 2013) was used to fill-in all the gravity voids before performing the gridding procedure. The fill-in procedure was achieved by: (1) determining a mask area using 25-km-radius circles centred on each of the land and marine gravity measurements, (2) inverting the mask area, (3) selecting those DTU13 grid nodes that overlap the inverted mask area, and (4) determining Δg_{RES} for the selected DTU13 fill-in gravity points. Then, the tensioned spline algorithm, introduced by Smith and Wessel (1990), with the recommended tension parameter T = 0.25was applied to determine a 1' gravity anomaly grid (Δg_{RES}^{grid}) using the observed and selected DTU13 gravity anomalies.

Finally, the residual Faye gravity anomaly grid, required as the input for Stokes' integral, was "reconstructed" by adding the Bouguer plate reduction to the grid nodes of Δg_{RES}^{grid} (Featherstone and Kirby 2000)

$$\Delta g_{RESFaye}^{grid} = \Delta g_{RES}^{grid} + 2\pi G \rho H^{grid}$$
(5)

where the height H was derived from the DEM.

The use of Faye gravity anomalies causes a change in the gravity potential due to the shifting of the topographic masses. Therefore, the so-called co-geoid is computed when using Faye anomalies. In order to determine the geoid, the indirect effect has to be determined and added to the computed co-geoid. This procedure is described in the next section.

2.2.2 Computing Step: Stokes' Integral

Stokes' integral is applied to determine geoid undulations from Δg quantities and it is given by (Heiskanen and Moritz 1967)

$$N(\varphi,\lambda) = \frac{R}{4\pi\rho} \iint_{\sigma} \Delta g S(\psi) \, d\sigma \tag{6}$$

where *R* is the mean radius of the Earth, $S(\psi)$ is the so-called Stokes' kernel and σ is the integration domain defined by a spherical cap of angular radius ψ_0 .

In this study, Eq. (6) was calculated using the multiband spherical FFT approximation technique (Sideris and Forsberg 1991) implemented in the SPFOUR software. The following central-latitude bands were selected for the computation: -20.00° , -29.25° , -38.50° , -47.75° and -57.00° . Moreover, the Wong and Gore (1969) modification to Stokes' kernel, which removes the low-degree Legendre polynomials that distort the long-wavelength signal of the geoid when integrating over a spherical cap, was adopted. The low harmonics were completely removed up to degree 120 and then linearly tapered to degree 130.

2.2.3 Restoring Step

The restore step of the RCR technique is achieved by adding the N_{GGM} and N_{IND} terms to the Stokes' integral result (i.e. N_{RES}). In this study, N_{GGM} was determined by Eq. (3) using the GEOCOL17 software together with GOCO05S GGM complete to degree and order 280.

Regarding N_{IND} , it can be split into the primary indirect effect (PITE) and the secondary indirect topographical effect (SITE). In this paper, the SITE was neglected due to the fact that the correction is less than 1 mm (Vaníček et al. 1999).

The PITE was calculated using the planar approximation introduced by Wichiencharoen (1982).

3 Gravity Data and Pre-processing

About 700,000 land and marine gravity observations that covered the area of interest (i.e. from latitude 20-57°S and from longitude 52-76°W) were collected from many public agencies and universities, including IGN; Instituto de Física de Rosario, Instituto de Geociencias Básicas, Aplicadas y Ambientales de la Universidad de Buenos Aires; Laboratorio de Geofísica Aplicada y Ambiental de la Universidad Nacional de Tucumán; Departamento de Física de la Universidad Nacional del Sur; Technische Universität Berlin; U.S. National Geospatial-Intelligence Agency; Bureau Gravimétrique International; Instituto Brasileiro de Geografia e Estatística; U.S. National Oceanic and Atmospheric Administration; Japan Agency for Marine-Earth Science and Technology; Marine Geoscience Data System; Rolling Deck to Repository; and British Antarctic Survey.

These observations, which are referred to the IGSN71 gravity system, were collected during the last 70 years using different types of methods and gravimeters, and therefore, the accuracy in most cases is uncertain. For this reason, the dataset was validated using three methods: (1) shipborne gravity observations were compared with oceanic gravity grids derived from satellite altimetry (independent of the ship-track gravity surveys), (2) observed gravity anomalies were compared with those derived from a satellite-only GGMs (Featherstone 2009), and (3) the least-squares collocation (LSC) method was used to detect gross errors in the gravity spatially-correlated database (Tscherning 1991).

The first method was applied using the Sandwell et al. (2014) 23.1-version altimeter-derived gravity anomaly grid. Table 1 shows the statistics of the differences between the retained marine gravity anomalies and the Sandwell's model.

The large differences appreciated in Table 1 mainly correspond to points in coastal areas, where the satellite altimetry data is not reliable.

The second method was achieved using the GOCO05S GGM complete to degree and order 280. The differences between the GGM and the observed gravity anomalies

Table 1 Statistics of the differences between the shipborne gravityanomalies and Sandwell's 23.1 version gravity model from 489,653measurements [Unit: (mgal)]

Min	Max	Mean	STD
-121.8	116.1	5.2	18.0

Table 2 Statistics of the differences between the observed gravityanomalies and the GOCO05S GGM from 560,656 measurements [Unit:(mgal)]

Min	Max	Mean	STD
-268.9	240.5	-10.6	33.3

were plotted on a map using level contours, where the existence of any deep holes or steep spikes indicated the existence of suspicious observations that were removed. Table 2 shows the statistics of the differences between the retained observed gravity anomalies and the GOC005S GGM.

The third method was applied using the EGM2008 GGM complete to degree and order 2159 and a gravity estimated error variance equal to 1 and 5 mgal for land and marine observations respectively.

A total of 560,656 gravity points were left after the quality check (Fig. 1) that were used to determine the required reduced gravity anomaly grid.

4 GEOIDEAR Validating and Fitting

The accuracy of GEOIDEAR was assessed using 1904 colocated GPS-levelling benchmarks (Fig. 2), whose ellipsoidal and orthometric heights were referred to the POSGAR 2007 geodetic reference frame and the 2014 Argentinean vertical datum respectively. GEOIDEAR was also fitted to these points by determining a trend surface using the fourparameter Helmert model (Iliffe et al. 2003)

$$N_{GPS-levelling} - N_{GEOAR} = a_1 \cos \varphi \cos \lambda + a_2 \cos \varphi \sin \lambda + a_3 \sin \varphi + a_4 + \varepsilon$$
(7)

where ε is the residual estimated by LSC. The covariance function was determined using the second-order Gauss-Markov model, in which an apriori error of 5 cm and a correlation length of 25 km were assumed.

Moreover, EGM2008-derived (Pavlis et al. 2008, 2012) geoid undulations were also computed for the 1904 locations and compared to GPS-levelling derived undulations. Table 3 shows the statistics of the differences between the GPS-levelling geoid undulations and those derived from GEOIDEAR and EGM2008 (complete to degree and order 2159).

Given that the published ARG05 and GAR Argentinean geoid models accuracies were defined using 539 and 393 points respectively, they were not comparable with the GEOIDEAR. Therefore, the geoid undulations derived from the 1904 co-located GPS-levelling benchmarks were used to determine the new accuracies of the ARG05 and GAR. Table 4 shows the differences between the GPS-levelling



Fig. 1 Land and marine gravity observations



Fig. 2 GPS-levelling measurements

Table 3 Statistics for the differences between the GPS-levelling and GEOIDEAR derived geoid undulations [Unit: (m)]

Model	Min	Max	Mean	STD
EGM2008	-0.98	1.67	0.62	0.31
GEOIDEAR (not fitted)	-0.91	1.86	0.69	0.27
GEOIDEAR (fitted)	-0.31	0.29	0.00	0.04

Table 4 Statistics for the differences between the GPS-levelling and the previous Argentinean geoids (i.e. ARG05 and GAR) [Unit: (m)]

Model	Min	Max	Mean	STD
ARG05	-0.04	5.32	1.92	0.59
GAR	-2.13	1.75	-0.09	0.29

geoid undulations and those derived from ARG05 and GAR.

Finally, the relative accuracy of the fitted GEOIDEAR model was determined for baselines within 500 km using the 1904 GPS-levelling benchmarks. Results show that 91% of the relative differences are less than ± 0.1 m and 98% of the differences are under ± 0.2 m. Figure 3 shows the relative differences for those baselines within 500 km (i.e.

351,582 baselines), which indicates that the results are not correlated with the location, and therefore, the relative precision of the new fitted geoid model can be considered homogenous.

The results show that GEOIDEAR (Fig. 4) outperforms EGM2008, GAR and ARG05 over Argentina. This is a consequence of the improved GGM's long-wavelength accuracy, the DEM's resolution reduction and corrections (e.g. voids elimination), and the use of the enhanced and recent dataset available (i.e. gravity, spirit-levelling and GPS measurements). GEOIDEAR's (fitted) STD shows an improvement of 83% with respect to GAR and 92% with respect to ARG05.

5 Conclusions

A new $1' \times 1'$ Argentinean gravimetric geoid model called GEOIDEAR was determined. The results showed that the STD of the GEOIDEAR is 4 cm. However, due to the ellipsoidal and orthometric heights accuracies used to fit the geoid (i.e. 5–10 cm), the propagated GPS-levelling derived geoid



Fig. 3 Relative differences between GEOIDEAR and the co-located GPS-levelling points



Fig. 4 GEOIDEAR geoid model

undulations accuracy is considered 7–14 cm, and therefore, the fitted GEOIDEAR's accuracy is expected to be 10 cm.

The use of GOCE data together with the latest released DTU's gravity field model, SRTM's products and IGN's co-located GPS-levelling benchmarks significantly improved the accuracy of the Argentinean geoid model. However, there are still many gravity voids that should be covered with land or airborne gravimetry in order to develop an improved geoid model.

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